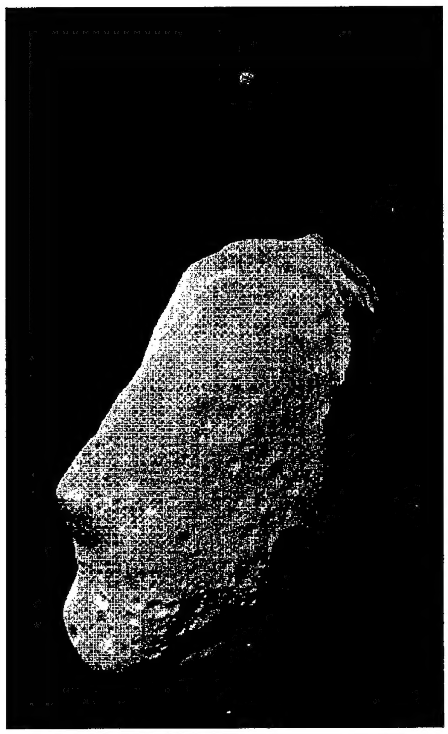
Predicting How Close Near-Earth Asteroids Will Come to Earth in the Next Five Years Using Only Kepler's Algorithm



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Master of Space Operations

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Abstract

*There are estimated to be over 150,000 near-earth asteroids in our solar system that are large enough to pose a significant threat to Earth. In order to determine which of them may be a hazard in the future, their orbits must be propagated through time. The goal of this investigation was to see if using only Kepler's algorithm, which ignores the gravitational pull of other planets, our moon, and Jupiter, was sufficient to predict close encounters with Earth. The results were very rough, and about half of the closest approaches were near the dates of those predicted by more refined models. The distances were in general off by a magnitude of ten, showing that asteroid orbits must be very perturbed by other planets, particularly Jupiter, over time and these must be taken into account for a precise distance estimate. A noted correlation was that the difference in the angular distance from the **I** vector was very small when the asteroid and Earth were supposed to be closest. In conclusion, using Kepler's algorithm alone can narrow down intervals of time of nearest approaches, which can then be looked at using more accurate propagators.*

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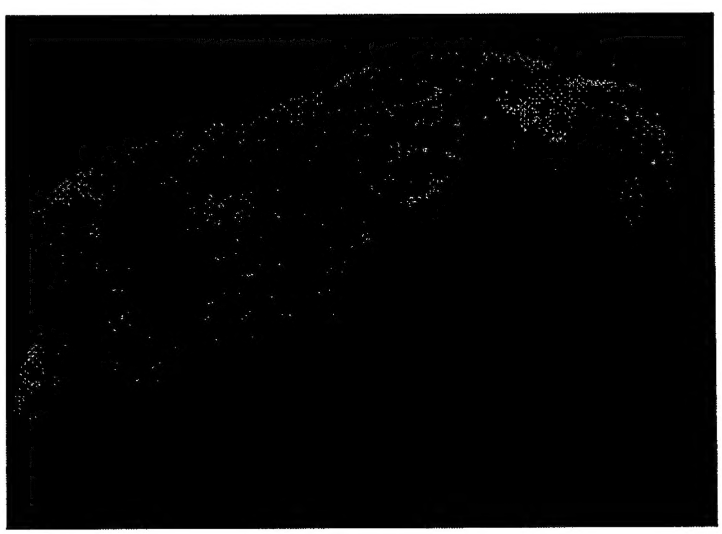
Background

An asteroid is a large chunk of rock, which can vary in size, color and composition. No one is certain how they were created, but asteroids are probably remnants of early solar system formation, or broken off parts of larger asteroids [1]. The main asteroid belt in our solar system consists of 95% of all known asteroids and exists between Mars and Jupiter, at a distance of 2.0 - 3.3 AUs from the Sun. Jupiter, being the heaviest of the planets, keeps this belt in its place. Its high gravitational attraction has prevented these numerous pieces of solar system rubble, whose total mass is less than the moon's [2], from becoming one body [3]. Instead, Jupiter's gravitational attraction has sped up many of the asteroids over time, causing them to collide with one another and form smaller pieces. Its pull forces many asteroids to change their orbits over years; they can end up shooting out of the solar system or into the inner, sometimes becoming near-Earth orbiters. Most rocks seem to be "in simple rotation with a fixed pole" [4] and have a period of 2 – 60 hours, the average being eight hours [5]. There are over 1 million that are larger than 1km, the biggest of them all being Ceres, which is 950 km in diameter. Luckily, it comes nowhere near Earth. Figure 1 [6] shows Ida, a 52-km main belt asteroid. The craters from other asteroids striking it are visible.

Figure 1



Asteroids and comets that have orbits which cross that of the Earth's are called near-Earth objects, or NEOs. Of these, approximately 10 percent are classed as long-period comets, which return every twenty years or more [6]. Asteroids whose orbits cross, or eventually will cross, that of the Earth's are termed near-Earth asteroids (NEA s) or Earth-crossing asteroids (ECA s). Those discovered so far range in size from 109 meters to 32 kilometers; there are doubtlessly asteroids much smaller than this, it is just very hard to uncover them. Most likely, there are some as huge as 3 to 5 km yet to be detected [7]. The largest ECA is 1627 Ivar, which is 8 km in diameter [8]. ECAs obviously have the greatest likelihood of all NEOs in colliding with the Earth. Although most of them will end up being ejected from the Solar System thanks to Jupiter's pull, some will strike a planet. One-third of these will hit Earth. About 250 ECAs have been discovered, but it has been estimated that there are thousands more. Approximately 1000-4000 ECAs are bigger than 1 kilometer (the minimum global catastrophe threshold – see Introduction), 5,000 - 20,000 larger than 500 meters, 150,000 - 1 million larger than 100 meters, and 10 million - 1 billion bigger than 10 meters [6]. Asteroids smaller than 10 meters are of little significance since they burn up passing through the Earth's atmosphere if they happen to run into it. Figure 2 [9] on the next page is a plot of the orbits of the hundred largest known ECAs, or only 5% of all known ECAs. It shows how congested the area around Earth can be. The dot in the center represents the Sun and Mercury's orbit is the dotted line closest to it. The outermost dotted line is Mars and Venus and Earth's are hard to see because the ECA orbits obscure them.

The near-Earth category of asteroids contains many different types; some primarily metal, others stony minerals, and still others composed of primitive early solar system matter [6]. They come from the main-belt asteroids "through collisional fragmentation and chaotic dynamics" [8]. But ECAs don't have to originate from the asteroid belt; they can also be

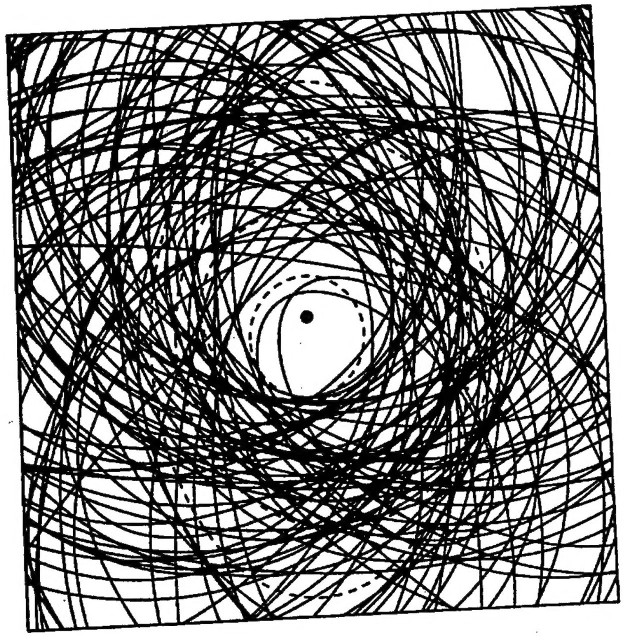
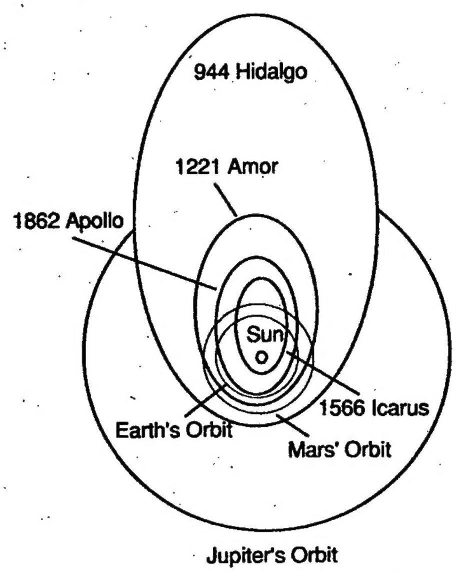


Figure 2

comets that can no longer produce a tail. After persistent close approaches to the Sun, these comets have burned up all volatile material [5]. ECAs also vary in spectral type, or brightness. Astronomers think that most of the brighter, larger ECAs, which are easier to spot, have already been discovered.

Within the ECA group, there are four divisions; Aten, Amor, Apollo and Arjuna. Aten asteroids have periods of less than one year and the majority of their orbit is within the Earth's, although they can cross parts of the Earth's orbit. Amor asteroids frequently cross Mars' orbit and approach ours. They may approach Earth as their orbits change over time, but currently present little threat. Apollos have their perihelion within the Earth's orbit, so that they frequently cross it, and a period longer than a year. They are most liable to collide with us. Figure 3 [6] on the next page compares the orbits of 1862 Apollo and 1566 Icarus, which are Apollos, and 1221 Amor, which is an Amor, to that of Earth. Finally, Arjuna asteroids have circular orbits within the Earth's and are smaller than 100 meters in size. This is a fairly new group, founded in 1994, and some claim these asteroids should instead be classed as Atens [6]. Scientists have

Figure 3



ascertained that around 65 percent of the ECAs are Apollos, 25 percent Amors and 10 percent Atens [6].

In the past, astronomers determined the orbits and classes of asteroids using a 0.46-m Schmidt telescope [10]. But 0.9-meter telescopes, such as the one used by the Spacewatch program, are replacing these. The Arizona-based Spacewatch tends to find around two to three new near-Earth objects per month. Spacewatch is a project that specifically sets out to track as many ECAs as possible and is the only program in the world with this sole goal. A charge-coupled device (CCD) camera is used with an exposure time of 2.5 minutes. Any object that moves shows up as a streak on the film and is picked out by a computer [11]. In Figures 4 [12] and 5 [1] the asteroid in each shows up as a band. Either a long-exposure image (Fig. 4) or its negative (Fig. 5) can be used. A 0.9-meter telescope scans the same region of sky in half-hour intervals three times. In order to have accurately measured positions, a newly discovered object should be reobserved on another night. Only then should the preliminary orbit be determined.

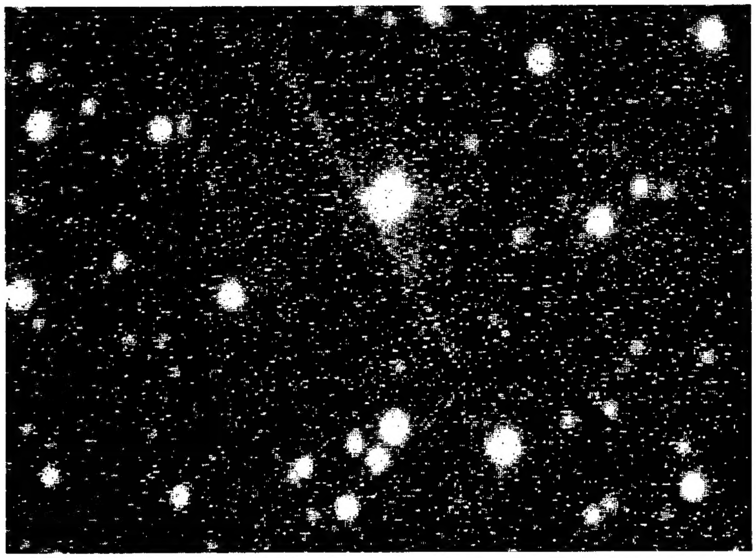


Figure 4

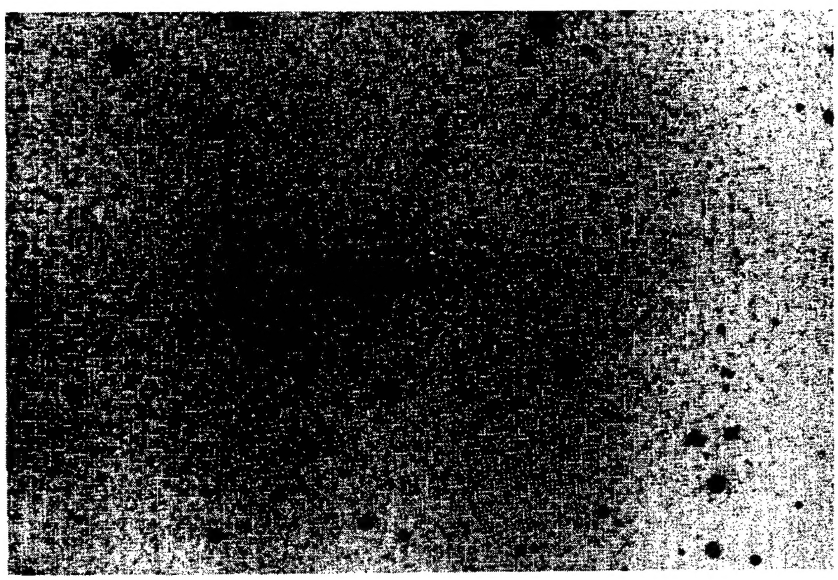


Figure 5

Astronomers have missed some asteroids, such as 1993 KA2, until they were very close. 1993 KA2 was 5 to 11 meters across and passed Earth less than halfway to the moon, 140,000 km or 0.0009 AUs, before it was noticed [6]. Many are missed because they are relatively small or approach from a sunward direction, making it near impossible for telescopes to pick them up [5].

The threat of asteroids was a hot topic last month when Spacewatch's James Scotti unearthed the perhaps-menacing asteroid 1997 XF11 [13]. At first, he forecast that it was on a straight collision course with Earth in 2028, soon changing that to coming within one-tenth of the distance to the moon. But he can't take into account any unexpected perturbations that may occur in the next few years and bring the asteroid even closer. There may be some sort of random gravitational perturbation such as a shock wave from a supernova or a collision with another asteroid. This demonstrates how hard it is to accurately predict a close encounter.

Once forecasts are as precise as possible, we can be on the lookout for a close approach and prepare for a potential collision. We can take steps to stop what could be a world disaster by investigating possible solutions such as nuclear deflection or fragmentation of the asteroid using pyrotechnic charges. This is why collision forewarning is so important; if a 1-km rock is hurtling our way it does not have to mean the end of the world.

Introduction

In order that we are aware of which asteroids may approach Earth dangerously close and be able to prepare possible defenses, the asteroid orbits must be determined. This can become quite involved and complicated. Perturbations on the orbits by Jupiter, Mars, the earth and even the moon must be considered. Most studies include forces from all of the other planets. Each orbit's line of apsides tends to advance slightly, particularly for orbits with small semimajor axis and large eccentricities [14]. My goal in this investigation is to predict which asteroids might threaten Earth in the next five years using only the simple Kepler algorithm, which propagates true anomalies, and thus orbits, through time. I want to see if it is possible to get a rough idea of when a potentially threatening asteroid may approach the Earth without taking into account the gravitational pull of the other planets and line of apsides advancement. I have the results from models that did include every perturbation with which to compare my calculations. I will assume a "close" approach to be less than 0.1 AUs [15], or 39 times the distance to the moon, because there is a risk at this distance that a slight change to an asteroid's orbit would bring it even nearer to Earth. The size of the asteroids I look at also establishes whether or not they are going to be hazards.

It is generally agreed [9] that the minimum global catastrophe threshold for asteroids is 1-km. If one this size were to strike Earth, there would be a severe greenhouse effect, 500km/hr plus winds, Tsunamis (large tidal waves), acid rain, triggered volcanism, darkness and cold. Most of these would last several months. A minimum of 25% of the world's population would be expected to die [9]. A smaller 100-meter asteroid would be expected to kill 1 million people; most from Tsunamis, the rest from the impact. Because of the number of people an asteroid might kill, we have more of a chance of dying as a result of one hitting Earth than of dying in a

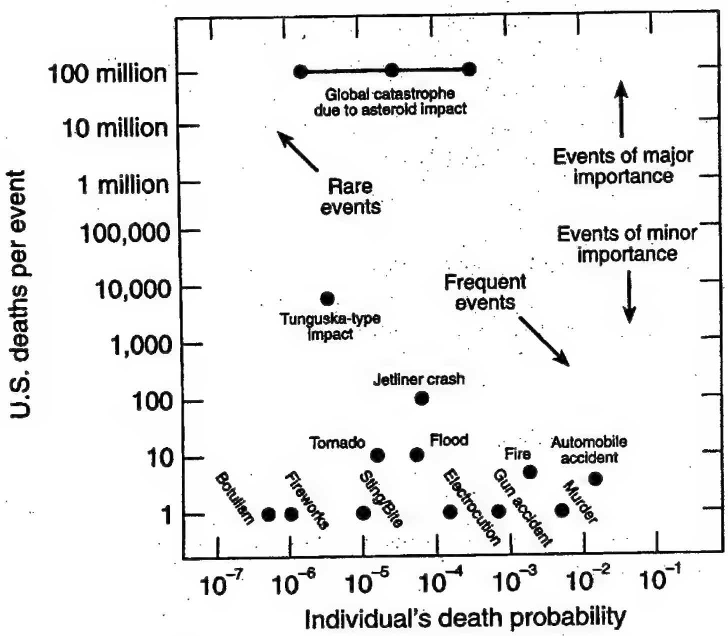


Figure 6 [16]

plane crash, or 1 in 10,000 (see Figure 6). So I am only going to look at asteroids larger than 0.1-km, which would be expected to significantly effect us if they landed here.

Finally, the maximum effect Jupiter could have on any given asteroid will be calculated, to see if it would change my results significantly, particularly for those asteroids that pass close to it.

Specific Topics of Review

As mentioned before, in this study I use Kepler's algorithm to propagate orbits. The positions of particular asteroids as well as that of the Earth are predicted in time intervals of ten days for five years. A smaller time period would take far more calculations and a longer one might miss a close encounter. Looking at more than five years would mean the gravitational effects I am ignoring would take on a greater significance. At the chosen times, I computed the distance between the Earth's position vector (to the Sun) and the asteroid's position vector.

The asteroids I investigated included those that have already been predicted to pass near the Earth in the next five years by NASA (Appendix I). Their predictions, which gave the Julian date and distance of closest approach, certainly took into account any movement of the line of apsides and any gravitational forces from other bodies, unlike my two-body problem. I then compared my results with theirs. I also randomly picked a few other NEAs to see what their closest approaches may be.

To begin with, I needed the orbital elements of the asteroids and of the Earth as inputs for the Kepler algorithm. For the asteroids, I utilized a chart from the Internet (Appendix II). It gives the accurate orbital elements of semi-major axis (a) in AUs, eccentricity (e), inclination (i) in degrees, longitude of ascending node (Ω), argument of perigee (ω) and true anomaly (M), all in degrees. They are given at JD 2449800 (March 23, 1995) for all asteroids discovered up until April 1995. Their orbits were determined after several sightings of each asteroid, using the sun as the focus. Usually three observations of an asteroid's position vector are sufficient [17] to get orbital elements, but sometimes many more, up to hundreds, are needed because the accuracy of the measurements is not good enough. For the Earth, I used orbital elements at JD2000 (Jan 1, 2000) [22].

As the Earth and asteroids move through time, their true anomalies change. The true anomaly ν is the angle from the vector **e**, which points to the perihelion, to the position vector **R** (see Figure 6). Other orbital elements, such as inclination and longitude of ascending node, change very slightly over time. I calculated i for the Earth over a time period of a year and the change in it was around a hundredth of a degree. However, since I am only dealing with five years and the orbits are rather large (unlike satellites orbiting Earth), I assumed the changes of all but the true anomalies to be negligible.

I came up with a C program (Appendix III) that would perform all the calculations I needed, taking in the orbital elements of an asteroid and coming up with an answer for the distance between that asteroid and the Earth at a particular point in time. Firstly, I wrote the function *Anomaly* to propagate the true anomalies of the asteroid and Earth. This was accomplished using the algorithm KepEqtnE [21], which required inputs of the eccentricity and the mean anomaly. Before the mean anomaly was sent to the function, it had to be converted to radians and calculated for the initial start date, which I chose to be April 1, 1998 (JD 2450905). This meant the asteroid Ms had to be propagated forwards and the Earth's backwards, using:

$$M_{new} = M + n^*\Delta t$$

Where n = mean motion = $\sqrt{(\mu / a^{\wedge}3)}$ (a was given in the charts in AUs & I changed it to km)
(μ is the gravitational parameter for the Sun)

and Δt = the time change in seconds.

Once both mean anomalies were set for the correct date, they were sent to the *Anomaly*, which performed the following calculations:

$$\text{Let initial } E_n = M_{new} \qquad (1)$$

$$\Rightarrow E_{n+1} = E_n + (M - E_n + e^*\mathrm{Sin}(E_n)) / (1 - e^*\mathrm{Cos}(E_n)) \qquad (2)$$

Where: M = mean anomaly (comes from true anomaly)
E = eccentric anomaly, e = eccentricity

The above Newton's Method involves repeating equation (2) until the difference between E_{n+1} and E_n is very small (I took a difference of 0.00001). The true anomaly ν can be worked out from the final E_{n+1} using the AnomalyToν [21] algorithm and making sure it is in the same hemisphere as E_{n+1}.

Before working out the R vector of each body, of which the true anomaly is one input, I computed the total angle of each body relative to the Sun's **I** vector and compared them. Figure 7 shows the geometry:

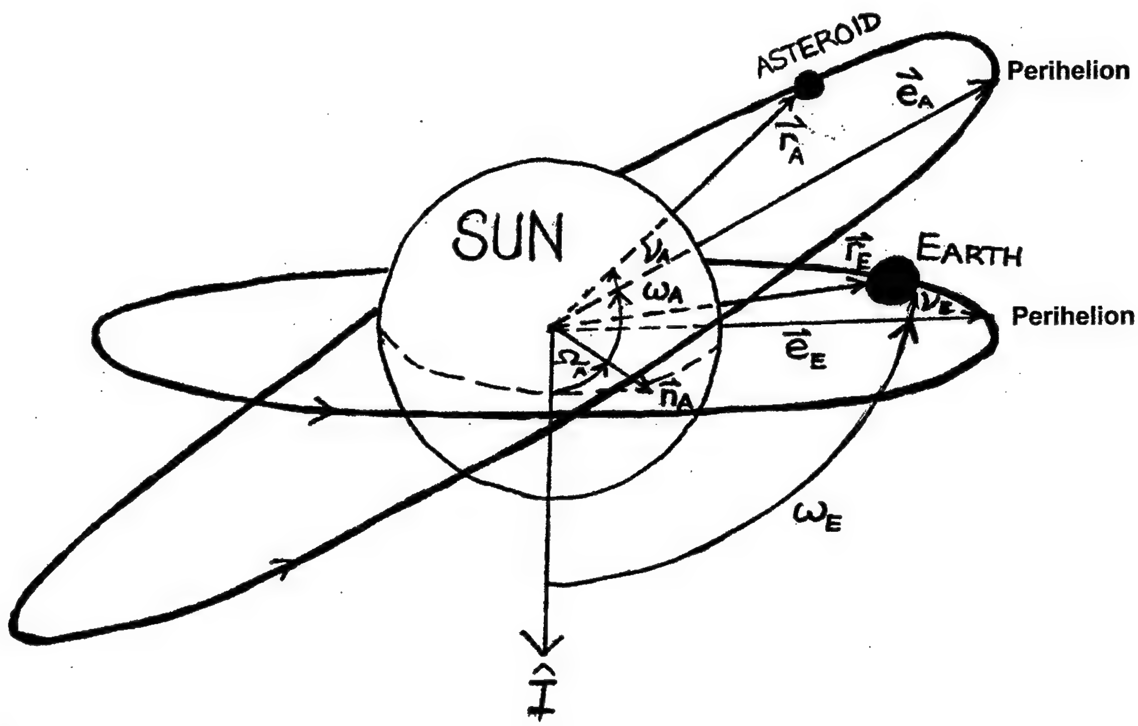


Figure 7

I added up the node (Ω), argument of perigee (ω) and true anomaly (ν) for the asteroid and for the Earth at the specific time. I then took the difference between the two sums. My guess was that the closest approaches are most likely to occur when the difference between the angles is rather small. At first I was going to use angle difference as a way to weed out dates during which there was a large angle difference (greater than 90°). I wouldn't even bother to calculate the

difference in R vectors on those days because I would expect it to be much larger than on days when the angle difference was small. Although this ought to be the case when the orbits are orientated as they are in Figure 7, it may not be true when the orbits have considerable dissimilar inclinations to the Sun, so all angle differences must be looked at. They were included in the printout of the results.

The next step after finding the true anomaly and angle difference was to work out the separate R vectors. This was accomplished using the RANDV algorithm [21], and my function *PKepler* in the program (Appendix III) includes it. It requires inputs of a, e, and ν. The semiparameter must be worked out using:

$p = a\,(1 - e^2)$ after a has been converted to AUs.

The equations returned the R vectors in the PQW, and not IJK, frame, but it didn't matter which frame I worked in since I was only looking for the magnitude of the difference between the Rs. Finally, my *PKepler* function subtracted the elements of the two R vectors and took the result's magnitude. This was continued for each chosen JD, the M being propagated by ten days each time, or a Δt of 864,000 seconds. The process carried on for a total time of five years, which in ten-day intervals is 183 iterations.

Since C programs can often work incorrectly, I worked out the answers by hand for a given JD and came up with the same angles and same differences in R's. To make sure the Kepler propagator was actually propagating as it should, I took Earth's orbital elements at a given JD and found out the separate elements of its R vector at that time. I then used a Δt of 365 days (one orbit) to propagate M, and looked at the new R. It was the same, suggesting that there were no errors in the propagator.

The results, an example of which is in Appendix IV, include the magnitude of the difference in R's (called 'R'), the difference in angles and the Julian date that they occurred. The name of the asteroid in question is the name of the file at the top of the printout. For each individual asteroid, I changed the basic program. The orbital elements had to be changed for every different program, obviously. I would start by having the program print out all distances less than 1 AU. If there were none, I would rerun it at 1.5 AU, etc. If there were many at 1 AU, I would reduce the R to 0.75AU or less. I wanted about a page of output for each asteroid. At first, I was going to reduce the JD interval around a close approach to narrow the prediction down to a day, but I could tell that my results were too rough for that to serve any purpose. I could only go as accurate as ten days, if that. Appendix IV also shows how R changes cyclically with time; it goes steadily up and down as the asteroid moves away from, and then towards, the Earth.

In Table 1 are the results for the asteroids chosen by NASA to approach closely in the next five years, along with my results. As can be seen, my distances for all but a couple of the asteroids are off by a magnitude of ten. This is probably because Jupiter, with its strong gravitational pull, was not taken into account. I shall look at that later. But I predicted six of the eleven asteroids to have their closest approach within 40 days of the "true" Julian date, and 1991RB matched the JD exactly. Forty days is a fairly small interval in the total time of an asteroid's orbit.

It would have been nice to also look at the new asteroid 1997 XF11, the one in recent newspapers [13] and mentioned in the background section, but the orbital elements were not yet available.

Asteroid	My closest determined approach (AU)	Determined JD (to the nearest 10 days)	Actual closest approach (AU)	JD	Diameter (km)
1991 RB	0.588	2451075	0.0401	2451075	0.6
1989 UR	0.752	2451665	0.0800	2451146	1
1992 SK	0.469	2450945	0.0560	2451264	1
1991 JX	1.965	2452515	0.0500	2451332	0.8
4486 Mithra	1.787	2451745	0.0466	2451771	3
4179 Toutatis	0.836	2451825	0.0739	2451849	3.3
1991 VK	0.019	2452315	0.0718	2452291	1.5
4660 Nereus	0.094	2452325	0.029	2452295	0.8
5604 1992 FE	0.873	2451595	0.0768	2452448	2
1991 BN	1.438	2452555	0.0775	2452593	0.4
1990 SM	0.419	2451595	0.0747	2452688	2

Table 1 [15]

Table 2 shows the minimum angle difference over the five years for each asteroid with Earth, and its JD. There seems to be a strong correlation between the JD of the smallest angle and the NASA-predicted JD of closest approach. *Ten* of the eleven asteroid JDs now match those from NASA, seven of those exactly or within five days. But for some of these small angles I got rather large R-values. This seems to suggest that predicting how close an asteroid shall get depends more on the calculated angular difference, not the difference in Rs, at least for my method. If I used an all-inclusive program, the gravitational pull of other bodies must change the asteroid's R vector significantly, whereas the angular difference remains the same (small for close approaches, which would make sense).

Asteroid	Least Angle Difference (°)	JD	NASA-predicted JD of closest approach
1991 RB	0.802	2451075	2451075
1989 UR	0.524	2451175	2451146
1992 SK	0.289	2451275	2451264
1991 JX	0.346	2451335	2451332
4486 Mithra	1.002	2451775	2451771
4179 Toutatis	0.326	2451855	2451849
1991 VK	0.275	2452295	2452291
4660 Nereus	0.373	2452295	2452295
5604 1992 FE	0.895	2451885	2452448
1991 BN	3.438	2452595	2452593
1990 SM	14.500	2452715	2452688

Table 2

I also made calculations on other asteroids I randomly picked from Appendix II:

Asteroid	Closest approach (AUs)	JD of closest approach	Least Angle Difference (°)	JD of least angle difference
<u>ATENS</u>				
1992BF*	0.134	2452085	64.904	2452605
1954XA	0.030	2452095	74.955	2452725
Aten	0.093	2452035	129.689	2451665
<u>APOLLOS</u>				
1988TA*	0.180	2452275	67.762	2452015
1987OA	0.767	2451055	0.468	2451055
Orpheus*	0.151	2452365	0.543	2450975
Heracles	0.307	2452365	2.733	2452315
1995EK1	0.025	2452375	42.275	2451115
<u>AMORS</u>				
Taranis	0.618	2451205	68.597	2451055
1994AW1*	0.671	2452735	50.173	2452345
1990BA	0.929	2451065	0.032	2452465

Table 3

Those asteroids marked with a star (*) in Table 3 have been predicted by NASA to come within 0.1AUs of the Earth in the next 20 years. The minimum distances from Earth are probably more accurate, or at least their magnitude should be since they're not supposed to come within 0.1AUs. These asteroids may not have approached Jupiter as closely as those in Table 1 and so their orbits did not change as much. The least angle difference is in general a lot higher, and since I speculated that a small difference suggests a close approach for my method, this would be correct.

If I would have had more time, I could have included the effects of Jupiter on the asteroids' orbits and calculated how much its pull would have changed the semimajor axis and thus closest approach to Earth. I did look at the maximum acceleration possible due to Jupiter for all asteroids in Table 1. I used the equation:

$a = \mu / r^2$, where a = acceleration (AU/TU^2)
μ = Sun's gravitational parameter = 1 AU^3/TU^2
r = *minimum* possible distance between the asteroid and Jupiter.

This should give me an idea of the magnitude of the acceleration at the closest approach of the asteroid and Jupiter. The closest approach was calculated by subtracting the two semimajor axes, and may actually be larger because I assumed the two bodies were in the same plane. Jupiter's semimajor axis is 5.2026 AUs. Table 4 on the next page gives results.

Also, severe changes in the minimum distance between the asteroid and Earth "occur mainly for the asteroids with revolution periods close to one-third that of Jupiter," [20] when their orbits are likely to change. Table 5 gives the periods of Jupiter and the Table 1 asteroids, as well as the fraction of Jupiter's period for each. Periods were worked out using:

$P = 2\pi\sqrt{(a^3/\mu)}$ where P = the period in TUs
a = the semimajor axis in AUs
μ = the Sun's gravitational parameter = 1 AU^3/TU^2

Asteroid	Maximum acceleration due to Jupiter (AU/TU^2)
1991 RB	0.0710
1989 UR	0.0588
1992 SK	0.0640
1991 JX	0.1389
4486 Mithra	0.1110
4179 Toutatis	0.1384
1991 VK	0.0886
4660 Nereus	0.0725
5604 1992 FE	0.0547
1991 BN	0.707
1990 SM	0.1078

Table 4

Asteroid	Period (TU)	Fraction of Jupiter's Period (TU)
JUPITER	74.561	-
1991 RB	10.973	0.147
1989 UR	7.053	0.095
1992 SK	8.765	0.118
1991 JX	25.120	0.337*
4486 Mithra	20.520	0.275*
4179 Toutatis	25.076	0.336*
1991 VK	15.728	0.211
4660 Nereus	11.424	0.153
5604 1992 FE	5.608	0.075
1991 BN	10.887	0.146
1990 SM	19.901	0.267*

Table 5

Those fractions marked with a '*' are close to one-third of Jupiter's period and so should be significantly affected by it. These results correlate with the acceleration due to Jupiter at a minimum distance – the same asteroids with periods near one-third of Jupiter's period also have higher accelerations in Table 4. The most significantly affected rocks are 1990SM, 1991JX, Mithra and Toutatis. This would explain why, in Table 1, 1991JX's JD was off by so much and its minimum distance was so large, and why 1990SM's JD was off by so much. Jupiter affects their orbits significantly, and a far better study could have been conducted if it had been included.

Discussion of Limitations

The main limitations in this investigation are obvious: I left out major gravitational forces. Had there been more time to do so, I could have added the effects due to Jupiter and Mars, as described earlier. The effect of the moon could have been included, and even the pull of other large asteroids the asteroid in question may have passed close to. But this would have involved a program of monstrous proportions, in which all of the above mentioned orbits were propagated and the gravitational effects determined.

I assumed all orbital elements but the true anomaly remained constant over five years because the change is so small. Perhaps including the changes would have moved numbers slightly.

The list of orbital elements in Appendix II was compiled by propagating the orbits and treating Earth and moon perturbations separately. Also, every planet's perturbations were taken into account at each step and general relativistic equations of motion were used [7]. The "general relativistic advancement in the line of apsides" [7] was accounted for when the asteroid had a large eccentricity and small semimajor axis. The accuracy of the orbital elements in the given tables may not be accurate enough. Over the past 20 years there have been slight changes to all of the orbital elements as more sightings have been recorded, giving a more accurate value. So over the next few years, it may be that the orbital element values I used will change slightly.

A problem I encountered when reading about how orbits were determined by astrophysicists was that many astronomical terms were used, such as "relativistic equations of motion". These have to do with how time travel might affect an asteroid's motion, slowing it down or speeding it up. Being an engineer, I had little understanding of such concepts, let alone

how to calculate them. It was mentioned that using ‘nonrelativistic equations’ of motion (which I did) would introduce a significant error.

I looked at only a sample of asteroids, and they were non-randomly picked. I used asteroids that had already been predicted to come close to Earth so I might check if I achieved the same results as NASA. The others I chose because I knew they *weren't* supposed to come close (“close” being within 0.1 AUs of the Earth).

I had plans to narrow the time interval, to get the day of closest approach, rather than the nearest 10 days. But my answers are so rough anyway that there seemed to be little point in narrowing down the time; the answer would just be very inaccurate. Of course, the NASA reference close approaches I used could be slightly wrong too, due to inaccuracy in the orbital elements or some factor being left out of the equations.

Finally, errors may have occurred if I mistyped the orbital elements into the program.

Discussion of Applications

The simple method I used can give a rough estimate of which asteroids might come closest to Earth and when. The key seemed to be looking at the minimum angle difference to find the time interval a closest approach may occur. The program is *not* accurate for predicting the distance of nearest approach.

A use for my program would be to find all times that the smallest angle difference is less than 15°. Then a more advanced program that takes longer and considers all other gravitational forces can use my time intervals to find a more accurate distance and time. This would save time because the more complicated program would not have to go through every possible date.

Not only could a program that works properly be used to predict collisions of asteroids with Earth, but also it could predict when they come close enough to Earth for us to be able to set up a rendezvous with them. A spacecraft could approach them to take photos or, in the future, perhaps we could mine the asteroids for their valuable minerals.

Conclusion

My investigation showed that it is not all that useful to only use Kepler's algorithm to predict how close asteroids may come to Earth. Using Kepler's algorithm, close approaches are quite inaccurate. However, it is possible to predict when they may be nearest, within 50 days or so. This is accomplished by looking at the angular difference between the asteroid and the Earth. If it is quite small (less than 15°), a close approach is likely. I would need to test this hypothesis on many more asteroids before I could be sure that it is true. For now, I would say that my program could give a rough estimate of the time period, thus narrowing down the dates to look at with a more advanced program. Including the gravitational effects of Jupiter may have improved my results dramatically.

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Appendix I

NASA Ames Space Science Division

Predicted Close Approaches

The following are some of the predicted close asteroid and comet approaches through the year 2020, supplied by Don Yeomans of the NASA/Caltech Jet Propulsion Laboratory.

NOTE: CA = closest approach distance to the Earth in Astronomical Units (150 million km)

Comets and asteroids passing within 0.1 AU of the Earth through 2020

Object	Date (TBD)	CA Distance	Absolute Magnitude	RA	DEC
1991 JX	1995 06 9.098	.0341	18.5	278	37
2063 Bacchus	1996 03 31.670	.0678	16.4	230	59
1991 CS	1996 08 28.419	.0620	17.5	53	-2
4197 1982 TA	1996 10 25.639	.0846	14.5	289	72
3908 1980 PA	1996 10 27.860	.0613	17.4	2	32
1991 VE	1996 10 29.543	.0853	19.0	296	-66
4179 Toutatis	1996 11 29.953	.0354	15.4	204	-22
1991 VK	1997 01 10.695	.0749	17.0	287	-18
6037 1988 EG	1998 02 28.914	.0318	18.7	77	-28
1991 RB	1998 09 18.475	.0401	19.0	170	-46
1989 UR	1998 11 28.689	.0800	18.0	4	-18
1992 SK	1999 03 26.265	.0560	17.5	28	41
1991 JX	1999 06 2.819	.0500	18.5	291	12
4486 Mithra	2000 08 14.365	.0466	15.4	112	-69
4179 Toutatis	2000 10 31.186	.0739	15.4	218	-21
1991 VK	2002 01 16.498	.0718	17.0	289	-24
4660 Nereus	2002 01 22.512	.0290	18.3	287	-13
5604 1992 FE	2002 06 22.264	.0768	17.0	157	-47
1991 BN	2002 11 14.726	.0775	20.0	325	19
1990 SM	2003 02 17.275	.0747	16.5	59	40
1991 JX	2003 05 20.681	.0922	18.5	301	-6
1990 OS	2003 11 11.448	.0250	20.0	194	40
1989 QF	2004 02 4.267	.0748	18.0	22	42
4179 Toutatis	2004 09 29.567	.0104	15.4	218	-60
1988 XB	2004 11 21.965	.0728	17.5	164	-1
1992 BF	2005 03 3.695	.0630	19.0	129	-62
1993 VW	2005 04 24.904	.0862	16.5	137	-24
1992 UY4	2005 08 8.424	.0402	17.5	359	12

Appendix II

Catalogue of Near Earth Asteroids: orbits of known Atens/Apollos/Amors as of 1995 March 21. Semimajor axes (a) are in astronomical units, angles are in degrees, and estimated diameters are in kilometers. The final column gives the spectral type if available. Data supplied by David Tholen of the University of Hawaii.

TABLE 1: ATENS

	Desig	a	e	i	Node	Peri	M	Epoch	Diam	Type
2062 Aten	1976 AA	0.9666	0.1826	18.93	107.99	147.82	299.11	2449800.5	1	S
2100 Ra-Shalom	1978 RA	0.8320	0.4365	15.75	170.24	355.95	104.33	2449800.5	4	C
2340 Hathor	1976 UA	0.8438	0.4499	5.85	210.93	39.83	3.31	2449800.5	0.5	CSU
3362 Khufu	1984 QA	0.9894	0.4686	9.91	151.97	54.84	345.56	2449800.5	1	
3554 Amun	1986 EB	0.9737	0.2804	23.35	358.01	359.34	308.42	2449800.5	3	
3753	1986 TO	0.9977	0.5147	19.81	125.70	43.62	67.79	2449800.5	5	
5381 Sekhmet	1991 JY	0.9474	0.2959	48.97	57.88	37.41	177.08	2449800.5	2	
5590	1990 VA	0.9853	0.2793	14.19	215.72	34.35	305.83	2449800.5	0.4	
5604	1992 FE	0.9270	0.4054	4.78	311.43	82.36	234.57	2449800.5	2	
	1995 CR	0.9057	0.8678	4.00	342.33	322.10	345.03	2449800.5	0.2	
	1994 XL1	0.6708	0.5263	28.19	252.02	356.49	27.61	2449800.5	0.3	
	1994 WR12	0.7540	0.4053	7.06	62.41	205.59	305.39	2449800.5	0.2	
	1994 TF2	0.9932	0.2836	23.75	174.64	349.62	29.72	2449800.5	0.6	
	1994 GL	0.6844	0.5019	3.64	196.46	179.06	69.10	2449800.5	0.06	
	1993 VD	0.8766	0.5493	2.05	2.25	253.51	348.33	2449800.5	0.2	
	1993 DA	0.9353	0.0937	12.40	328.53	353.96	300.17	2449800.5	0.02	
	1992 BF	0.9079	0.2710	7.25	314.94	336.28	82.03	2449800.5	0.6	
	1991 VE	0.8905	0.6638	7.20	61.49	193.33	93.67	2449800.5	0.6	
	1989 VA	0.7286	0.5947	28.79	224.95	2.79	42.66	2449800.5	1	
	1989 UQ	0.9152	0.2649	1.28	178.03	14.62	272.91	2449800.5	0.6	
	1954 XA	0.7775	0.3451	3.91	188.77	58.73	120.76	2449800.5	0.6	

Catalogue of Near Earth Asteroids: orbits of known Atens/Apollos/Amors as of 1995 March 21. Semimajor axes (a) are in astronomical units, angles are in degrees, and estimated diameters are in kilometers. The final column gives the spectral type if available. Data supplied by David Tholen of the University of Hawaii.

TABLE 2: APOLLOS

	Desig	a	e	i	Node	Peri	M	Epoch	Diam	Type
1566 Icarus	1949 MA	1.0780	0.8267	22.88	87.47	31.20	10.06	2449800.5	2	
1620 Geographos	1951 RA	1.2455	0.3354	13.33	336.65	276.74	191.54	2449800.5	2.3	S
1685 Toro	1948 OA	1.3670	0.4360	9.37	273.70	126.95	18.24	2449800.5	12	S
1862 Apollo	1932 HA	1.4711	0.5598	6.35	35.27	285.58	44.34	2449800.5	1.5	Q
1863 Antinous	1948 EA	2.2600	0.6062	18.41	346.84	266.91	280.57	2449800.5	2	SU
1864 Daedalus	1971 FA	1.4609	0.6146	22.17	6.09	325.43	126.42	2449800.5	3	SQ
1865 Cerberus	1971 UA	1.0801	0.4669	16.09	212.35	325.13	204.71	2449800.5	1	S
1866 Sisyphus	1972 XA	1.8933	0.5391	41.16	63.01	292.97	224.27	2449800.5	9.6	
1981 Midas	1973 EA	1.7761	0.6500	39.83	356.48	267.68	81.87	2449800.5	4	
2063 Bacchus	1977 HB	1.0779	0.3495	9.41	32.63	55.05	92.76	2449800.5	2	
2101 Adonis	1936 CA	1.8738	0.7648	1.34	350.02	42.30	42.70	2449800.5	0.7	
2102 Tantalus	1975 YA	1.2901	0.2985	64.01	93.70	61.60	13.25	2449800.5	3.3	
2135 Aristaeus	1977 HA	1.5997	0.5031	23.04	190.74	290.63	341.09	2449800.5	1	
2201 Oljato	1947 XC	2.1759	0.7106	2.51	76.34	95.81	254.33	2449800.5	2	
2212 Hephaistos	1978 SB	2.1681	0.8335	11.77	27.75	208.36	82.17	2449800.5	5	SG
2329 Orthos	1976 WA	2.4024	0.6587	24.41	168.84	145.75	352.75	2449800.5	4	
3103 Eger	1982 BB	1.4060	0.3546	20.93	129.22	253.79	31.51	2449800.5	3	
3200 Phaethon	1983 TB	1.2713	0.8901	22.10	264.87	321.83	23.14	2449800.5	5.2	F
3360	1981 VA	2.4645	0.7430	21.74	244.81	60.58	183.33	2449800.5	2	
3361 Orpheus	1982 HR	1.2091	0.3225	2.68	189.11	301.53	303.54	2449800.5	0.6	
3671 Dionysus	1984 KD	2.1952	0.5429	13.62	81.76	203.63	105.31	2449800.5	2	
3752 Camillo	1985 PA	1.4136	0.3023	55.54	147.33	312.21	154.91	2449800.5	3	
3757	1982 XB	1.8350	0.4465	3.87	74.50	16.77	331.11	2449800.5	0.5	S
3838 Epona	1986 WA	1.5047	0.7018	29.28	235.06	49.43	219.51	2449800.5	3	
4015 Wilson-Harrington	1979 VA	2.6418	0.6219	2.78	270.14	91.07	216.92	2449800.5	4	CF
4034	1986 PA	1.0599	0.4441	11.17	157.44	296.44	222.07	2449800.5	1	
4179 Toutatis	1989 AC	2.5154	0.6361	0.47	128.30	274.08	212.59	2449800.5	3.3	
4183 Cuno	1959 LM	1.9809	0.6369	6.76	295.17	235.22	321.88	2449800.5	5	
4197	1982 TA	2.2972	0.7727	12.20	9.50	119.23	182.29	2449800.5	5	
4257 Ubasti	1987 QA	1.6471	0.4684	40.70	168.65	278.89	165.20	2449800.5	2	
4341 Poseidon	1987 KF	1.8357	0.6787	11.86	107.55	15.47	83.01	2449800.5	3	
4450 Pan	1987 SY	1.4418	0.5865	5.51	311.52	291.38	165.58	2449800.5	1.5	
4486 Mithra	1987 SB	2.2012	0.6626	3.04	81.94	168.47	138.47	2449800.5	3	
4544 Xanthus	1989 FB	1.0420	0.2502	14.14	23.44	333.62	66.04	2449800.5	1.5	
4581 Asclepius	1989 FC	1.0226	0.3571	4.91	179.85	255.00	354.62	2449800.5	0.3	
4660 Nereus	1982 DB	1.4897	0.3602	1.41	313.97	157.96	91.02	2449800.5	0.8	
4769 Castalia	1989 PB	1.0631	0.4832	8.88	325.04	121.22	334.57	2449800.5	1.6	
4953	1990 MU	1.6208	0.6569	24.42	77.42	77.38	167.94	2449800.5	5	
5011 Ptah	6743 P-L	1.6350	0.4999	7.39	10.38	105.36	124.94	2449800.5	1.5	
5131	1990 BG	1.4866	0.5700	36.38	109.83	135.69	242.65	2449800.5	6	
5143 Heracles	1991 VL	1.8338	0.7713	9.18	310.04	226.39	78.41	2449800.5	6	
5189	1990 UQ	1.5508	0.4778	3.58	134.80	159.37	148.71	2449800.5	1	
5496	1973 NA	2.4337	0.6382	68.02	100.38	118.24	272.29	2449800.5	4	
5645	1990 SP	1.3548	0.3874	13.51	45.23	47.96	255.08	2449800.5	2	
5660	1974 MA	1.7861	0.7628	37.94	301.85	126.71	216.91	2449800.5	3	
5693	1993 EA	1.2721	0.5853	5.05	96.60	258.56	336.98	2449800.5	1.5	
5731	1988 VP4	2.2632	0.6526	11.64	282.06	215.63	283.48	2449800.5	3	
5786 Talos	1991 RC	1.0814	0.8267	23.24	160.67	8.26	173.14	2449800.5	2	
5828	1991 AM	1.6977	0.6957	30.04	124.91	152.53	285.71	2449800.5	2	
6037	1988 EG	1.2690	0.4994	3.48	182.27	241.46	18.74	2449800.5	0.7	
6047	1991 TB1	1.4537	0.3521	23.46	5.57	103.55	292.54	2449800.5	1.5	
6053	1993 BW3	2.1474	0.5288	21.59	317.96	74.57	298.72	2449800.5	5	
6063	1984 KB	2.2157	0.7643	4.85	169.25	336.49	113.98	2449800.5	4	
6239	1989 QF	1.1513	0.4127	3.93	344.15	239.50	250.24	2449800.5	0.9	
	1995 EK1	2.2509	0.7746	8.81	355.03	296.53	342.08	2449800.5	1	
	1995 DW1	1.0405	0.4380	15.10	348.26	326.88	265.09	2449800.5	0.2	
	1995 DV1	3.1257	0.6915	3.69	171.63	283.97	14.05	2449800.5	0.07	
	1995 CS	1.9400	0.7753	2.61	135.06	252.05	34.39	2449800.5	0.04	
	1995 BL2	1.2315	0.4943	23.08	311.72	348.22	252.60	2449800.5	1	
	1994 XM1	2.0884	0.5688	4.11	76.35	40.66	24.37	2449800.5	0.01	
	1994 XG	1.5719	0.4900	11.32	231.10	46.14	157.53	2449800.5	1	
	1994 XD	2.3590	0.7291	4.34	96.96	247.73	48.66	2449800.5	0.6	
	1994 VH8	1.6355	0.4428	3.37	37.78	314.14	84.29	2449800.5	0.01	
	1994 UG	1.2258	0.2457	4.49	11.73	225.91	247.95	2449800.5	0.2	
	1994 RC	2.2678	0.6018	4.72	345.45	284.34	75.84	2449800.5	0.6	
	1994 RB	2.4883	0.6394	26.62	338.94	52.47	41.21	2449800.5	0.1	
	1994 PC1	1.3457	0.3277	33.50	117.29	47.53	273.04	2449800.5	2	
	1994 PM	1.4775	0.7523	17.94	139.38	303.27	92.69	2449800.5	1	
	1994 NE	2.0360	0.6045	27.53	104.29	246.20	72.70	2449800.5	0.4	
	1994 LX	1.2614	0.3464	36.90	110.65	349.01	24.35	2449800.5	4	
	1994 GV	2.0300	0.5228	0.45	19.31	154.30	125.40	2449800.5	0.01	

1994 GK	1.9609	0.6064	5.66	14.67	111.58	142.34	2449800.5	0.06
1994 FA	1.7358	0.4157	13.04	355.09	154.66	170.31	2449800.5	0.04
1994 ES1	1.4117	0.5897	0.94	352.34	279.11	188.61	2449800.5	0.008
1994 EU	1.3725	0.2749	6.44	350.96	145.77	250.26	2449800.5	0.03
1994 EK	1.9992	0.6067	5.76	333.54	98.01	159.90	2449800.5	0.4
1994 CN2	1.5732	0.3949	1.43	98.93	247.89	345.40	2449800.5	2
1994 CK1	1.8977	0.6200	4.40	328.11	27.67	247.78	2449800.5	1.5
1994 CJ1	1.4908	0.3258	2.31	171.61	64.92	161.22	2449800.5	0.2
1994 CC	1.6370	0.4169	4.63	268.11	24.63	63.62	2449800.5	1
1994 CB	1.1491	0.1450	18.25	310.04	288.40	242.82	2449800.5	0.2
1994 AH2	2.5255	0.7114	9.63	163.68	24.82	80.74	2449800.5	2
1993 XN2	2.1173	0.5356	25.38	59.06	312.89	172.33	2449800.5	2
1993 WD	1.0066	0.2666	63.46	55.89	132.30	10.01	2449800.5	2
1993 VW	1.6949	0.4843	8.68	230.58	280.93	174.42	2449800.5	2
1993 VB	1.9098	0.5197	5.06	145.23	322.63	164.63	2449800.5	0.5
1993 VA	1.3559	0.3912	7.26	132.60	336.35	281.97	2449800.5	1.5
1993 UC	2.4380	0.6626	25.99	165.41	322.95	103.06	2449800.5	3
1993 UA	2.0201	0.5244	4.58	26.48	330.11	186.57	2449800.5	0.04
1993 TZ	2.0235	0.5633	4.16	202.96	231.23	166.07	2449800.5	0.02
1993 QA	1.4767	0.3161	12.63	146.09	323.25	198.77	2449800.5	1
1993 PC	1.1543	0.4743	4.15	336.85	168.12	289.61	2449800.5	1
1993 PB	1.4232	0.6064	40.84	315.32	212.21	79.63	2449800.5	2
1993 KA2	2.2273	0.7745	3.18	238.78	261.44	213.05	2449800.5	0.006
1993 KH	1.2338	0.3110	12.80	53.86	293.57	46.54	2449800.5	0.6
1993 KA	1.2552	0.1974	6.05	235.17	341.90	125.65	2449800.5	0.02
1993 HP1	1.9915	0.5104	8.00	36.37	152.25	251.92	2449800.5	0.02
1993 HD	1.4322	0.6633	5.74	201.77	252.55	78.03	2449800.5	0.1
1993 HC	1.9888	0.5069	9.39	200.85	306.32	266.26	2449800.5	0.3
1993 GD	1.1022	0.2380	15.45	200.87	201.92	24.57	2449800.5	0.3
1993 FA1	1.4262	0.2886	20.46	186.67	343.60	68.99	2449800.5	0.03
1993 BX3	1.3953	0.2809	2.79	174.99	289.77	123.00	2449800.5	0.3
1993 BW2	1.3352	0.3061	21.91	120.50	287.37	185.43	2449800.5	1
1992 YD3	1.1661	0.1372	27.04	273.63	173.75	284.16	2449800.5	0.02
1992 UY4	2.6543	0.6198	2.83	308.41	37.43	211.13	2449800.5	1
1992 TB	1.3418	0.4622	28.31	185.02	5.92	5.91	2449800.5	1
1992 SY	2.2087	0.5503	8.02	5.62	114.98	221.63	2449800.5	1
1992 SK	1.2485	0.3249	15.31	8.36	233.48	14.91	2449800.5	1
1992 QN	1.1908	0.3593	9.58	355.40	202.11	92.97	2449800.5	2
1992 LC	2.5193	0.7049	17.84	61.28	89.64	267.31	2449800.5	4
1992 JD	1.0347	0.0316	13.54	221.92	285.87	338.93	2449800.5	0.03
1992 JB	1.5564	0.3598	16.07	217.81	306.75	203.71	2449800.5	1
1992 HF	1.3907	0.5617	13.30	212.91	128.05	219.90	2449800.5	0.3
1992 HE	2.2402	0.5722	37.36	26.62	262.57	295.62	2449800.5	6
1992 DU	1.1598	0.1748	25.05	337.28	121.63	207.37	2449800.5	0.04
1992 CC1	1.3914	0.3749	36.88	348.61	21.90	70.72	2449800.5	4
1992 BC	1.4135	0.3484	14.21	122.82	77.07	277.34	2449800.5	0.6
1991 XA	2.2690	0.5704	5.28	76.17	309.11	357.59	2449800.5	0.06
1991 WA	1.5752	0.6425	39.65	66.07	241.73	275.36	2449800.5	2
1991 VK	1.8436	0.5059	5.41	294.37	173.25	101.30	2449800.5	1.5
1991 VH	1.1363	0.1437	13.91	138.82	206.99	320.12	2449800.5	1.5
1991 VG	1.0269	0.0491	1.44	73.51	24.15	38.74	2449800.5	0.007
1991 VA	1.4288	0.3516	6.52	36.97	313.41	17.16	2449800.5	0.02
1991 TF3	2.0414	0.5303	14.04	6.02	303.20	86.12	2449800.5	0.6
1991 TB2	2.3977	0.8352	8.62	296.41	195.66	317.29	2449800.5	1.5
1991 TU	1.4068	0.3306	7.55	192.73	222.09	5.56	2449800.5	0.008
1991 TT	1.1929	0.1605	14.75	191.77	218.11	209.31	2449800.5	0.02
1991 RB	1.4502	0.4839	19.53	358.88	68.71	333.61	2449800.5	0.6
1991 LH	1.3520	0.7305	52.06	280.16	203.92	253.90	2449800.5	1.5
1991 JX	2.5190	0.5990	2.31	212.16	64.47	337.11	2449800.5	0.8
1991 JW	1.0383	0.1183	8.71	53.41	301.76	116.55	2449800.5	0.5
1991 GO	1.9594	0.6618	9.66	24.30	88.58	177.46	2449800.5	0.6
1991 EE	2.2460	0.6244	9.76	168.46	115.03	30.78	2449800.5	1
1991 DG	1.4271	0.3629	11.15	179.63	63.05	89.94	2449800.5	0.6
1991 CB1	1.6870	0.5946	14.56	316.85	345.53	191.78	2449800.5	1
1991 CS	1.1229	0.1646	37.10	156.23	249.26	235.76	2449800.5	1
1991 BN	1.4426	0.3979	3.44	268.46	80.54	237.31	2449800.5	0.4
1991 BB	1.1863	0.2725	38.47	294.34	322.81	333.44	2449800.5	2
1991 BA	2.1652	0.6734	2.12	118.25	71.86	98.58	2449800.5	0.008
1991 AQ	2.2210	0.7771	3.21	342.06	239.69	78.62	2449800.5	1.5
1990 UO	1.2341	0.7582	29.34	205.06	332.95	7.23	2449800.5	0.3
1990 UN	1.7093	0.5277	3.66	7.62	97.08	325.95	2449800.5	0.08
1990 UA	1.7212	0.5519	0.96	102.09	203.87	9.71	2449800.5	0.5
1990 TG1	2.4849	0.6929	9.06	204.37	33.28	114.25	2449800.5	4
1990 SS	1.7029	0.4748	19.39	359.43	115.74	310.26	2449800.5	0.6
1990 SM	2.1567	0.7754	11.56	137.28	105.91	168.03	2449800.5	2
1990 OS	1.6691	0.4590	1.10	347.40	20.03	33.69	2449800.5	0.4
1990 MF	1.7468	0.4556	1.86	209.87	113.89	0.37	2449800.5	0.8
1990 HA	2.5777	0.6932	3.88	184.43	307.89	80.94	2449800.5	1.5
1989 VB	1.8648	0.4608	2.13	38.34	329.60	51.13	2449800.5	0.4
1989 UR	1.0801	0.3563	10.34	233.89	289.29	198.37	2449800.5	1

	1989 UP	1.8637	0.4732	3.86	52.79	17.21	33.00	2449800.5	0.3	
	1989 JA	1.7704	0.4841	15.23	60.93	231.80	152.06	2449800.5	1.5	
	1989 DA	2.1623	0.5433	6.44	349.08	138.73	334.08	2449800.5	1	
	1989 AZ	1.6457	0.4682	11.76	295.06	111.63	1.15	2449800.5	0.5	
	1988 XB	1.4674	0.4816	3.12	73.02	279.79	228.74	2449800.5	1	
	1988 TA	1.5408	0.4786	2.54	194.49	104.57	163.78	2449800.5	0.2	
	1987 OA	1.4962	0.5956	9.02	179.69	235.38	22.98	2449800.5	0.8	
	1986 JK	2.7960	0.6806	2.13	62.17	232.40	311.67	2449800.5	0.6	
	1984 QY1	3.5976	0.9390	17.85	144.72	335.79	188.67	2449800.5	11	
	1983 VB	1.8189	0.4752	12.05	247.70	115.36	244.46	2449800.5	0.8	
	1983 VA	2.6106	0.6926	16.24	76.81	11.67	240.77	2449800.5	2	
	1983 LC	2.6304	0.7089	1.52	159.07	184.67	259.64	2449800.5	0.6	
	1979 XB	2.2622	0.7141	24.85	85.14	75.64	162.17	2449800.5	0.6	
	1978 CA	1.1246	0.2148	26.11	160.63	102.12	30.27	2449800.5	1	S
	1950 DA	1.6840	0.5023	12.08	356.28	224.26	203.55	2449800.5	3	
Hermes	1937 UB	1.6463	0.6236	6.13	34.11	91.85	27.19	2449800.5	1	
	6344 P-L	2.6270	0.6309	4.47	180.50	237.09	30.16	2449800.5	0.2	
	5025 P-L	4.2089	0.8947	6.34	354.67	151.44	341.82	2449800.5	5	

Go to: Near Earth Asteroids

Catalogue of Near Earth Asteroids: orbits of known Atens/Apollos/Amors as of 1995 March 21. Semimajor axes (a) are in astronomical units, angles are in degrees, and estimated diameters are in kilometers. The final column gives the spectral type if available. Data supplied by David Tholen of the University of Hawaii.

TABLE 3: AMORS

	Desig	a	e	i	Node	Peri	M	Epoch	Diam	Type
433 Eros		1.4582	0.2229	10.82	303.71	178.60	161.38	2449800.5	20.	S
719 Albert		2.5840	0.5393	11.24	183.96	155.07	42.87	2449800.5	2	
887 Alinda		2.4928	0.5594	9.27	110.09	349.81	136.26	2449800.5	5.4	S
1036 Ganymed		2.6597	0.5379	26.63	215.11	132.27	76.91	2449800.5	41	S
1221 Amor	1932 EA1	1.9196	0.4353	11.89	170.83	26.30	229.59	2449800.5	1	
1580 Betulia	1950 KA	2.1946	0.4901	52.12	61.68	159.31	289.57	2449800.5	7.6	C
1627 Ivar	1929 SH	1.8632	0.3967	8.44	132.61	167.41	291.73	2449800.5	7.0	S
1915 Quetzalcoatl	1953 EA	2.5362	0.5741	20.46	162.36	347.90	177.59	2449800.5	0.5	SMU
1916 Boreas	1953 RA	2.2731	0.4494	12.83	340.20	335.27	54.42	2449800.5	3	S
1917 Cuyo	1968 AA	2.1513	0.5039	23.94	187.77	194.26	259.00	2449800.5	6	
1943 Anteros	1973 EC	1.4302	0.2559	8.70	245.73	338.20	287.25	2449800.5	2	S
1980 Tezcatlipoca	1950 LA	1.7096	0.3649	26.85	246.03	115.29	313.20	2449800.5	13	SU
2059 Baboquivari	1963 UA	2.6490	0.5264	11.00	200.43	191.17	102.17	2449800.5	2	
2061 Anza	1960 UA	2.2654	0.5365	3.76	207.07	156.42	40.91	2449800.5	3	TCG
2202 Pele	1972 RA	2.2900	0.5125	8.78	169.71	217.20	166.86	2449800.5	2	
2368 Beltrovata	1977 RA	2.1046	0.4138	5.24	287.05	42.24	273.73	2449800.5	3	SQ
2608 Seneca	1978 DA	2.4906	0.5816	15.34	168.91	33.90	118.21	2449800.5	1	S
3102 Krok	1981 QA	2.1523	0.4475	8.41	171.70	154.31	109.06	2449800.5	1	QRS
3122 Florence	1981 ET3	1.7685	0.4227	22.17	335.53	27.55	155.48	2449800.5	6	
3199 Nefertiti	1982 RA	1.5743	0.2837	32.96	339.41	53.32	90.86	2449800.5	3	S
3271	1982 RB	2.1024	0.3949	25.00	158.35	158.66	51.69	2449800.5	2	
3288 Seleucus	1982 DV	2.0325	0.4573	5.93	218.11	349.24	168.15	2449800.5	3	S
3352 McAuliffe	1981 CW	1.8784	0.3694	4.77	106.87	15.59	175.37	2449800.5	3	
3551 Verenia	1983 RD	2.0932	0.4866	9.50	173.29	193.06	287.25	2449800.5	1	V
3552 Don Quixote	1983 SA	4.2340	0.7141	30.78	350.02	316.59	121.87	2449800.5	18	D
3553 Mera	1985 JA	1.6446	0.3203	36.76	231.95	288.85	282.42	2449800.5	2	
3691	1982 FT	1.7743	0.2839	20.37	348.24	234.61	161.62	2449800.5	5	
3908	1980 PA	1.9241	0.4587	2.17	261.18	125.72	146.52	2449800.5	0.8	V
3988	1986 LA	1.5446	0.3166	10.77	229.30	86.62	171.68	2449800.5	0.8	
4055 Magellan	1985 DO2	1.8201	0.3263	23.24	164.29	154.11	253.29	2449800.5	3	V
4401 Aditi	1985 TB	2.5762	0.5676	26.79	23.31	67.08	83.58	2449800.5	3	
4487 Pocahontas	1987 UA	1.7302	0.2966	16.40	197.55	173.72	99.68	2449800.5	1	
4596	1981 QB	2.2392	0.5188	37.12	153.80	248.30	354.00	2449800.5	2	
4688	1980 WF	2.2347	0.5143	6.41	241.03	212.91	96.71	2449800.5	0.5	QU
4947 Ninkasi	1988 TJ1	1.3696	0.1682	15.65	214.84	192.74	345.90	2449800.5	0.8	
4954 Eric	1990 SQ	2.0020	0.4481	17.47	358.12	52.01	188.99	2449800.5	12	
4957 Brucemurray	1990 XJ	1.5654	0.2189	35.01	254.29	97.46	115.89	2449800.5	4	
5324 Lyapunov	1987 SL	2.9591	0.6152	19.48	352.42	320.15	181.22	2449800.5	6	
5332	1990 DA	2.1635	0.4564	25.43	142.48	305.56	235.47	2449800.5	7	
5370 Taranis	1986 RA	3.3469	0.6318	19.01	177.19	161.11	142.58	2449800.5	5	
5587	1990 SB	2.3923	0.5483	18.09	189.88	86.19	110.92	2449800.5	7	
5620	1990 OA	2.1591	0.4222	7.84	128.31	152.95	177.25	2449800.5	1.5	
5626	1991 FE	2.1955	0.4543	3.86	172.87	231.07	168.09	2449800.5	4	
5646	1990 TR	2.1420	0.4375	7.90	13.61	335.37	162.24	2449800.5	5	
5653	1992 WD5	1.7942	0.3038	6.86	9.50	122.15	315.07	2449800.5	3	
5751	1992 AC	2.1045	0.4211	16.05	121.12	25.13	5.32	2449800.5	8	
5797	1980 AA	1.8924	0.4441	4.18	298.41	168.13	303.93	2449800.5	0.6	
5836	1993 MF	2.4438	0.5319	8.03	240.43	74.78	154.67	2449800.5	6	
5863	1983 RB	2.2220	0.5062	19.43	168.79	114.78	192.76	2449800.5	3	
5869 Tanith	1988 VN4	1.8121	0.3210	17.94	227.37	230.55	189.33	2449800.5	2	
5879	1992 CH1	1.6245	0.2893	21.57	145.27	355.45	183.08	2449800.5	1	
6050	1992 AE	2.2027	0.4363	6.40	87.90	284.53	17.30	2449800.5	3	
6178	1986 DA	2.8205	0.5823	4.29	64.41	126.87	321.62	2449800.5	4	
	1995 BK2	2.4417	0.5501	24.73	130.51	349.56	16.04	2449800.5	0.1	
	1995 BC2	1.9175	0.4304	5.02	328.01	81.19	55.76	2449800.5	2	
	1994 US	2.7182	0.5671	8.46	223.06	121.45	44.21	2449800.5	0.2	
	1994 TE2	2.3043	0.4548	5.70	198.07	182.14	46.33	2449800.5	0.2	
	1994 TA2	2.6781	0.5312	7.07	200.47	119.12	52.54	2449800.5	0.3	
	1994 TW1	2.5912	0.5768	36.03	2.92	62.19	17.94	2449800.5	5	
	1994 RH	2.2466	0.4414	18.92	330.99	91.81	23.17	2449800.5	3	
	1994 QC	1.3243	0.1179	13.87	161.92	94.08	197.80	2449800.5	0.6	
	1994 PN	2.3757	0.5400	46.05	112.54	233.88	40.75	2449800.5	2	
	1994 PC	1.5683	0.3173	9.45	123.89	256.52	75.95	2449800.5	2	
	1994 NK	2.3515	0.5387	5.67	119.54	128.48	82.34	2449800.5	0.4	
	1994 ND	2.1663	0.5160	27.19	102.14	227.88	68.20	2449800.5	1	
	1994 LW	3.1624	0.6195	23.02	240.44	54.41	41.52	2449800.5	2	
	1994 JX	2.7372	0.5793	32.68	52.23	192.21	65.48	2449800.5	1	
	1994 GY	2.6699	0.5318	12.46	33.43	189.95	72.14	2449800.5	2	
	1994 EF2	2.2919	0.5176	23.31	345.75	123.68	123.38	2449800.5	2	
	1994 BB	2.0222	0.4257	1.14	122.19	335.84	154.30	2449800.5	0.1	
	1994 AW1	1.1046	0.0752	24.09	289.74	37.13	149.00	2449800.5	2	

1994 AB1	2.8432	0.5915	4.52	66.49	342.39	104.86	2449800.5	2
1993 VC	2.7742	0.5330	3.20	241.94	177.04	102.41	2449800.5	0.3
1993 UD	1.3194	0.1942	22.78	24.46	254.66	68.33	2449800.5	0.4
1993 UB	2.2773	0.4602	25.02	30.83	20.78	138.90	2449800.5	2
1993 TQ2	1.9863	0.4198	6.04	13.01	77.24	158.63	2449800.5	0.4
1993 RA	1.9276	0.4186	5.70	171.27	265.28	158.66	2449800.5	0.8
1993 QP	2.3072	0.4693	7.24	296.70	46.55	159.58	2449800.5	1
1993 OM7	1.3398	0.2331	25.95	296.98	142.47	265.31	2449800.5	0.8
1993 MO	1.6261	0.2208	22.63	110.90	167.06	296.20	2449800.5	2
1993 HO1	1.9872	0.4165	5.90	22.23	104.98	290.34	2449800.5	2
1993 HA	1.2782	0.1442	7.73	182.76	263.47	234.13	2449800.5	0.4
1993 FS	2.2267	0.4247	10.13	178.73	20.82	211.32	2449800.5	0.4
1993 DQ1	2.0399	0.4911	9.97	313.03	344.41	144.45	2449800.5	2
1993 BU3	2.4061	0.5142	5.29	315.61	144.39	215.05	2449800.5	0.2
1993 BD3	1.6346	0.3748	0.88	312.96	168.86	12.77	2449800.5	0.02
1993 BD2	2.1230	0.3933	25.59	96.50	65.06	221.55	2449800.5	0.6
1992 UB	3.0658	0.5823	15.94	73.68	290.67	166.82	2449800.5	2
1992 TC	1.5657	0.2923	7.08	88.11	275.36	95.98	2449800.5	1
1992 SZ	2.1769	0.4599	9.27	3.74	314.60	292.98	2449800.5	0.4
1992 SL	1.6415	0.3339	8.59	0.41	344.50	75.71	2449800.5	1.
1992 OM	2.1935	0.4087	8.21	313.15	346.80	298.57	2449800.5	2
1992 NA	2.3910	0.5603	9.75	348.91	7.85	245.63	2449800.5	2
1992 LR	1.8310	0.4090	2.02	232.38	67.85	26.56	2449800.5	1
1992 JE	2.1895	0.4632	5.86	193.25	109.45	288.40	2449800.5	2
1992 BL2	1.6814	0.2300	36.86	297.15	23.40	311.09	2449800.5	4
1992 BA	1.3412	0.0676	10.48	139.64	107.17	255.11	2449800.5	0.3
1992 AA	1.9816	0.3897	8.29	102.15	354.30	54.79	2449800.5	2
1991 XB	2.9552	0.5867	16.29	249.74	172.00	233.78	2449800.5	1
1991 RJ2	2.2105	0.4279	8.91	171.39	150.33	36.09	2449800.5	0.6
1991 PM5	1.7194	0.2551	14.42	132.09	140.27	244.15	2449800.5	1
1991 OA	2.5081	0.5876	5.51	305.88	317.27	340.77	2449800.5	1
1991 NT3	1.8109	0.3041	13.86	286.78	292.75	226.14	2449800.5	5
1991 JG1	1.3732	0.1844	33.85	225.78	322.57	172.70	2449800.5	0.6
1991 JR	1.4032	0.2598	10.11	59.50	207.11	96.35	2449800.5	0.1
1991 FB	2.3675	0.5628	9.19	18.41	218.33	22.61	2449800.5	0.6
1991 FA	1.9790	0.4467	3.07	338.84	91.76	210.37	2449800.5	2
1991 DB	1.7163	0.4019	11.43	157.78	50.98	269.25	2449800.5	0.8
1990 VB	2.4433	0.5277	14.56	253.91	102.25	60.05	2449800.5	2
1990 UP	1.3253	0.1685	28.06	32.59	293.85	9.69	2449800.5	0.3
1990 SA	1.9579	0.4295	37.53	171.69	114.31	260.05	2449800.5	2
1990 KA	2.1987	0.4328	7.56	105.10	146.50	168.81	2449800.5	2
1990 BA	1.7403	0.3376	1.99	311.17	170.79	88.00	2449800.5	1
1989 RS1	2.3039	0.4817	7.18	174.03	180.86	205.91	2449800.5	1
1989 RC	2.3123	0.5138	7.38	139.68	181.09	213.16	2449800.5	1
1989 OB	2.6963	0.5594	7.91	289.02	71.72	84.97	2449800.5	2
1989 ML	1.2723	0.1364	4.37	103.83	183.10	350.60	2449800.5	0.5
1988 SM	1.6628	0.3433	10.92	0.38	312.92	30.25	2449800.5	1
1988 PA	2.1502	0.4077	8.21	161.74	136.95	41.55	2449800.5	2
1988 NE	2.1808	0.4424	9.93	253.53	354.81	42.48	2449800.5	0.8
1987 WC	1.3619	0.2336	15.83	51.27	308.10	261.24	2449800.5	0.5
1987 SF3	2.2541	0.5333	3.32	187.02	133.63	88.85	2449800.5	0.8
1987 QB	2.7927	0.5975	3.48	152.88	156.10	228.26	2449800.5	0.6
1987 PA	2.7172	0.5640	16.35	307.97	337.77	258.27	2449800.5	0.8
1986 NA	2.1260	0.4495	10.34	243.21	35.68	293.62	2449800.5	0.4
1985 WA	2.8345	0.6058	9.78	43.00	351.11	344.97	2449800.5	0.8
1983 LB	2.2865	0.4770	25.35	80.82	220.18	132.56	2449800.5	2
1982 YA	3.6952	0.6993	34.90	268.91	143.80	264.76	2449800.5	4
1977 VA	1.8642	0.3940	2.98	223.94	172.37	299.74	2449800.5	0.6
1977 QQ5	2.2256	0.4662	25.20	133.83	247.77	88.64	2449800.5	4
1972 RB	2.1499	0.4856	5.22	176.81	152.34	59.99	2449800.5	0.6
4788 P-L	2.6274	0.5501	10.98	176.92	97.14	69.97	2449800.5	2

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Appendix III

```
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// Creative Investigation program
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#include <stdio.h>
#include <math.h>
#include <iostream.h>
#include <fstream.h>

double TAnomaly(double, double);
double ETAnomaly(double, double);
double PKepler(double, double, double, double, double, double);


main()
{
    double Pi, e, M, node, peri, a, Ee, EM, Enode, Eperi, Ea;
    double Ev, v, n, En, EAngle, Angle, Total, Mnew, EMnew, R;
    int JD, x;

    Pi = 3.14159265359;

    ofstream out ("1987OA.txt");  // Name of file is name of asteroid

    e = 0.5956;         // Orbital elements for the asteroid in question
    M = 22.98;          // Degrees
    node = 179.69;      // Degrees
    peri = 235.38;      // Degrees
    a = 1.4962;         // AU s

    M = (M*Pi)/180;
    node = (node*Pi)/180;
    peri = (peri*Pi)/180;
    a = a*149597870.0;
    n = sqrt(132712428000/(pow(a,3)));

    Ee = 0.0167;                 // All of these for JD = 2451545 (JD2000)
    EM = 357.5291;               // Equals lamdaM - omegasqiggle = -2.4709 degrees
    Enode = 0.0000;              // All in degrees
    Eperi = 102.9373 - Enode;    // Equals  omegasquiggle - node
    Ea = 149597870.0;            // In km already

    EM = (EM*Pi)/180;            // JD = 2451545
    Enode = (Enode*Pi)/180;
    Eperi = (Eperi*Pi)/180;

    En = sqrt(132712428000/(pow(Ea,3)));

    Mnew = M + (n*95472000);     //Propagating M from 2449800 to 2450905
    EMnew = EM - (En*55296000); // And EM backwards to 2450905 (12 00, April 1, 1998)

    while (Mnew > (2*Pi))
        Mnew = Mnew - (2*Pi);
    while (EMnew < (0))
        EMnew = EMnew + (2*Pi);

    x = 0;
```

```
JD = 2450905;


while (x < 184)              //Want 183 iterations (10 day intervals for 5 years)
{
Ev = ETAnomaly (Ee, EMnew);
v = TAnomaly (e, Mnew);

Angle = node + peri + v;
EAngle = Enode + Eperi + Ev;

while (Angle < 0)
    Angle = Angle + (2*Pi);
while (Angle > (2*Pi))
    Angle = Angle - (2*Pi);
while (EAngle < 0)
    EAngle = EAngle + (2*Pi);
while (EAngle > (2*Pi))
    EAngle = EAngle - (2*Pi);

Total = EAngle - Angle;
if (Total < 0)
    Total = -Total;
if (Total > Pi)               // To get the smallest difference in true anomalies
    Total = 2*Pi - Total;
Total = (Total*180)/Pi;       // Put in degrees


if(Total < 15)                // Measure difference in r's IF angle between asteroid
{                             // and Earth is < 30 degrees

  R = PKepler(Ea, Ee, Ev, a, e, v);    // R is returned in AUs
 if(R < 1.0)                           // Only want R s less than 1 AU
 {out << "\n R is  " << R;
  out << "   Angle difference = " << Total;
  out << "  JD = " << JD;
 }

}

Mnew = Mnew + (n*864000);     //Propagating M's by 10 days
EMnew = EMnew + (En*864000);
while (Mnew > (2*Pi))
    Mnew = Mnew - (2*Pi);
while (EMnew > (2*Pi))
    EMnew = EMnew - (2*Pi);

JD = JD +10;
x = x+1;
}

out.close();
}



// Function finds Earth true anomaly at a given JD :
```

```
double ETAnomaly (double Ee, double EMnew)
{
   double Pi, E0, E1, Ev,term, sinv1, cosv1;
   int x;

   Pi = 3.14159265359;

   E0 = EMnew;
   x = 0;

    while( x < 1000)                              // Propagating for E
    {
     E1 = E0 + ( (EMnew - E0 + (Ee*sin(E0)) ) / (1 - Ee*cos(E0)) );
     if( E1 > E0)
     {    if (E1 - E0 < 0.00001)
               x = 1001;
     }
     else
     { if (E0 - E1 < 0.00001)
               x = 1001;
     }

     E0 = E1;
     x = x + 1;

    }

   term = sqrt(1 - pow(Ee,2));

   sinv1 = (term*sin(E1)) / (1 - Ee*cos(E1));
   cosv1 = (cos(E1) - Ee) / (1 - Ee*cos(E1));
   Ev = atan2(sinv1,cosv1);                   // Finding true anomaly & checking quadrants

   return Ev;
}




// Function finds asteroid true anomaly at a given JD :

double TAnomaly(double e, double Mnew)
{
   double E0, E1, v, Pi, term, sinv1, cosv1;
   int x;

   Pi = 3.14159265359;

   E0 = Mnew;
   x = 0;

    while( x < 1000)                              // Propagating for E
    {
     E1 = E0 + ( (Mnew - E0 + (e*sin(E0)) ) / (1 - e*cos(E0)) );
     if( E1 > E0)
     {    if (E1 - E0 < 0.00001)
               x = 1001;
     }
```

```
  else
  { if (E0 - E1 < 0.00001)
       x = 1001;
  }

  E0 = E1;
  x = x + 1;

 }

 term = sqrt(1 - pow(e,2));

 sinv1 = (term*sin(E1)) / (1 - e*cos(E1));
 cosv1 = (cos(E1) - e) / (1 - e*cos(E1));
 v = atan2(sinv1,cosv1);                   // Finding true anomaly & checking quadrants
 return v;
}




// Function finds the difference in  Earth and Asteroid r vectors (in the PQW frame)

double PKepler(double Ea, double Ee, double Ev, double a, double e, double v)
{ double p1, p2, Er1, Er2, Er3, r1, r2, r3;
  double R1, R2, R3, R;

  Ea = Ea / 149597870;                     // Change to AUs
  p1 = Ea*(1 - pow(Ee, 2));
  Er1 = (p1*cos(Ev)) / (1 + Ee*cos(Ev));   // The Earth's r vector
  Er2 = (p1*sin(Ev)) / (1 + Ee*cos(Ev));
  Er3 = 0;

  a = a / 149597870;                       // Change to AUs
  p2 = a*(1 - pow(e, 2));
  r1 = (p2*cos(v)) / (1 + e*cos(v));       // The asteroid's r vector
  r2 = (p2*sin(v)) / (1 + e*cos(v));
  r3 = 0;

  R1 = r1 - Er1;                           // Figure the difference in r's
  R2 = r2 - Er2;
  R3 = r3 - Er3;

  R = sqrt(pow(R1,2) + pow(R2,2) + pow(R3,2));   // Find the magnitude of R
  return R;
}
```

Appendix IV

```
R is  0.619971   Angle difference = 10.7427  JD = 2452165
R is  0.536488   Angle difference = 5.63706  JD = 2452175
R is  0.472445   Angle difference = 0.937095  JD = 2452185
R is  0.427186   Angle difference = 3.27842  JD = 2452195
R is  0.39748   Angle difference = 6.91406  JD = 2452205
R is  0.377872   Angle difference = 9.85718  JD = 2452215
R is  0.362097   Angle difference = 11.9804  JD = 2452225
R is  0.344581   Angle difference = 13.1506  JD = 2452235
R is  0.321292   Angle difference = 13.2485  JD = 2452245
R is  0.290002   Angle difference = 12.2052  JD = 2452255
R is  0.25027   Angle difference = 10.0547  JD = 2452265
R is  0.203286   Angle difference = 6.98881  JD = 2452275
R is  0.151515   Angle difference = 3.37894  JD = 2452285
R is  0.0980593   Angle difference = 0.275011  JD = 2452295
R is  0.0463143   Angle difference = 3.46354  JD = 2452305
R is  0.0193329   Angle difference = 5.79214  JD = 2452315
R is  0.0627334   Angle difference = 7.04303  JD = 2452325
R is  0.114805   Angle difference = 7.16175  JD = 2452335
R is  0.1726   Angle difference = 6.20442  JD = 2452345
R is  0.238753   Angle difference = 4.28346  JD = 2452355
R is  0.315456   Angle difference = 1.52905  JD = 2452365
R is  0.403969   Angle difference = 1.9326  JD = 2452375
R is  0.504691   Angle difference = 5.98921  JD = 2452385
R is  0.61737   Angle difference = 10.5455  JD = 2452395
```